ROBUST ESTIMATION FOR DETECTION OF FLATNESS DEFECTS

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Abstract

Shape verification is a common task in engineering, geodetic practice and rely on fitting the theoretical model describing the object in its actual shape. The theoretical model (e.g. plane) is defined in the design documentation, while the actual shape of the object is determined by points that are defined as a result of geodetic measurements. The shape verification is determined at the moment when the measurement was performed. The paper concerns the use of robust estimation methods to the study of flatness of the object measured by geodetic methods. Conducted studies involving the simulation of displacements (imperfections) of a known size and their detection by fitting a plane surface model using the weighted least squares method and robust estimation methods. The conducted research indicates the predominance of robust estimation methods to the classic method of least squares in the matter of resistance to incidental values of deviations from the plane surface that may have a source both in the actual imperfection and in the errors of the measurement nature.

Key words: robust estimation methods, modelling, least squares method, deformation

Introduction

A common task in geodetic practice is to control the geometry of an engineering object. Levelness, verticality or flatness of structural elements are checked (WYCZAŁEK et al., 2015; RINALDO et al., 2016). Measurements and their analysis are carried out both during construction and subsequent operation stages. As a result of geodetic measurements we get a set of points representing the shape of a given object. Level control is carried out for objects such as ceilings, floors, foundation slabs, as well as bridge and viaduct slabs. In some cases, there is a need for regulatory or repair work. Classical methods used in surface approximation of such an object are often sensitive to outliers or gross errors in the sets of points representing the shape. In order to minimize costs, labour consumption and working time, to better fit the object model, it would be advisable to use methods minimizing the impact of outliers on the final parameters of the estimated model.

The verticality control is performed for such objects as walls and elevations of buildings, elevator shafts, pillars' surfaces, bridge abutment. Prefabricated elements of the facade of large commercial and service buildings became popular in recent years. Their correct assembly requires verticality control of the walls on which they are installed. In the case of elevator shafts, verticality measurements usually concern the calculation of wall position deviations (with door openings) from the vertical position with reference to the wall on the lowest storey (MUSZYŃSKI, 2007; MUSZYŃSKI, 2008).

Failure to keep the vertical position of structural elements of buildings and structures affects their proper operation and safety. Also in the case of this type of work, it would be optimal to apply calculation methods that minimize residuals of the model obtained at individual points representing the object from the theoretical plane.

The measurement of the points' grid on the celling of settling building can be used to fit into the set of points of the best fitted plane and thus determine the building's inclination. Local shape anomalies may disturb the results of the object's inclination and it would be desirable to use the method suppressing the local geometrical imperfections reflected in the measurement results.

The classic least squares method used to minimize the sum of weighted deviations may turn out to be less effective than the robust estimation methods. Significant local deformations of the measured surface may cause false orientation of reference plane as well as raised values of residuals.

The paper attempts to test the appropriateness of using robust estimation methods in plane approximation. The wall of the building was measured and its shape was approximated. Obtained results were referenced to the classic method of least squares.

Robust estimation methods used in research

Huber Method

The Huber method was developed by Professor P. J. Huber (HUBER, 1964). In order to derive a robust M-estimator, he applied a probability density function consisting of two different functions. The centre of the set is ruled by normal distribution, while moving away from the centre it goes into Laplace, i.e. a particular form of exponential distribution. The weight function can be represented in the form (GÖTZELMAN et al., 2006):

$$w(e)=\begin{cases}1 & \text{for} \quad |e|\le k \\ \dfrac{k}{|e|} & \text{for} \quad |e|>k\end{cases} \tag{1}$$

In literature, we find many suggestions for choosing k parameter. Huber reports that its value adjusts the magnitude of robustness and states that a good choice is between 1 and 2. The control parameter k, which is responsible for determining the limits of the acceptable range for corrections, can be set, among others empirically. In this work, the parameter value for this method has been selected to achieve 95 % efficiency of the estimator with reference to standard normal distribution.

Hampel Method

The function responsible for modifying weights in the Hampel method is more extensive than in the Huber method and is presented as follows (HAMPEL, 1974; HOAGLIN et al., 1983):

$$w(e)=\begin{cases}1 & \text{for} \quad |e|\le a \\ \dfrac{a}{|e|} & \text{for} \quad a<|e|\le b \\ \dfrac{a\cdot(c-|e|)}{(c-b)\cdot|e|} & \text{for} \quad b<|e|\le c \\ 0 & \text{for} \quad |e|>c\end{cases} \tag{2}$$

The parameters a, b, c that appear in the form of functions define the limits of intervals in which the value of the correction may be found. In the literature, one can find the definitions that the Hampel function is an extension of the Huber function with two additional intervals introduced, i.e. to the left and right of the range acceptable for corrections in the Huber function (MUSZYŃSKI, 2007). In the literature, many suggestions for selecting control parameters can be found. In the paper (HOGG, 1979) we find that the reasonable choice is a = 1.7, b = 3.4, c = 8.5. These parameters have been accepted for calculations in this study. They provide 95% efficiency of the estimator with reference to standard normal distribution.

Danish Method

The sensitivity of the least-squares method to gross errors was reflected in the work of the Danish Geodetic Institute. The Danish method was introduced by T. Krarup, an outstanding Danish surveyor. He proposed it in 1967 and was used since then in the automatic search for outliers in the observation sets. The weighting function in this method is (KRARUP, 1980; WIŚNIEWSKI, 2009):

$$w(e)=\begin{cases}1 & \text{for} \quad e\in\langle -k;k\rangle \\ e^{-l(|e|-k)^{g}} & \text{for} \quad |e|>k\end{cases} \tag{3}$$

Parameters l and g should be chosen experimentally. Usually the value of l is chosen from the range from 0.01 to 0.1, while g = 2.

Linear Method

The method was proposed by Professor E. Osada in 2002 (OSADA, 2002). The idea of the linear method differs significantly from the other methods presented in this work. Assigning new weights to observations is based on the analysis of the values of corrections for observations after each iteration of alignment. The average errors of observations, used to construct weights, are subject to modification. The average error of a given observation is increased iteratively by the value of the excess of the standard deviation from the set of corrections. The first step of the iterative calculation process is weighting least squares. The method of weight modification is presented in the expression (MUSZYŃSKI, 2008; BANAŚ, 2017):

$$m_i^{(j+1)} = \begin{cases} m_i & for \quad \left|v_i^{(j)}\right| \le k_i^{(j)} \cdot m \\ m_i + \left|v_i^{(j)}\right| & for \quad \left|v_i^{(j)}\right| > k_i^{(j)} \cdot m_i \end{cases} \tag{4}$$

In the above formula $k_i^{(j)} = \sigma^{(j)} / m_i$ and i is the number of observations.

Geodetic inspection of wall flatness

The object to control was the wall in room W-21 in the Department of Integrated Geodesy, located on the campus of the State Higher School of Technology and Economics in Jarosław. The room in which the measurements were taken serves as a lecture hall for students and PWSTE employees. The wall was selected in such a way that a regular grid of squares could be displayed on it with the help of a multimedia projector.

Geometry of measured net of points and technique of surveying

In order to obtain the surplus observations necessary to control and obtain the assumed accuracy, it was decided to measure from two measurement stations. They were located at a distance of about 1m from the wall opposite the wall covered by the measurement (Fig. 1). The distance of the measuring stations from each other was 4.198m. This value was determined during the measurements. On the right side of the examined object there was a position marked as ST.1, while on its left a ST.2 station. Control points for locating stations were marked on the floor with a small cross made of permanent ink. To set the measuring equipment (total station, reflector) above the marked points, wooden tripods where used which ensured stability of the station. Before the measurement, the instrument and reflector were levelled and centred over the point. Surveying activities were performed with using Leica TS02 total station and Leica GRZ4 360 prism. From two stations in a room, angles and distances where measured to each of 112 points marked on the wall (Fig. 2). In the first, original survey, the points reflecting the actual shape of the wall were measured, i.e without intentionally simulated displacements.

Empirical tests of robust estimation methods

Preparing contaminated set of coordinates for numerical tests

The wooden blocks of rectangular shape were used to disturb the actual shape of the measured wall. Their task was to simulate displacements of chosen points. Blocks were covered with white self-adhesive paper. Small crosses where marked on them to allow precise aiming. These blocks have been carefully measured using a calliper to get to know their actual size, and hence the size of the marked imperfections. Then, before making the measurement, they were glued to the wall with a strong double-sided tape (Fig. 3). As displaced points, 9 nodes were adopted with the following numbers: 44, 45, 46, 67, 68, 69, 72, 73, 74. The dimensions of the blocks measured with the calliper are summarized in Table 1. The right-hand side of Fig. 3 presents a disturbed wall model with deformation of exactly known size.

Fig. 1. Localization of control points during measurements.
Source: Own study.

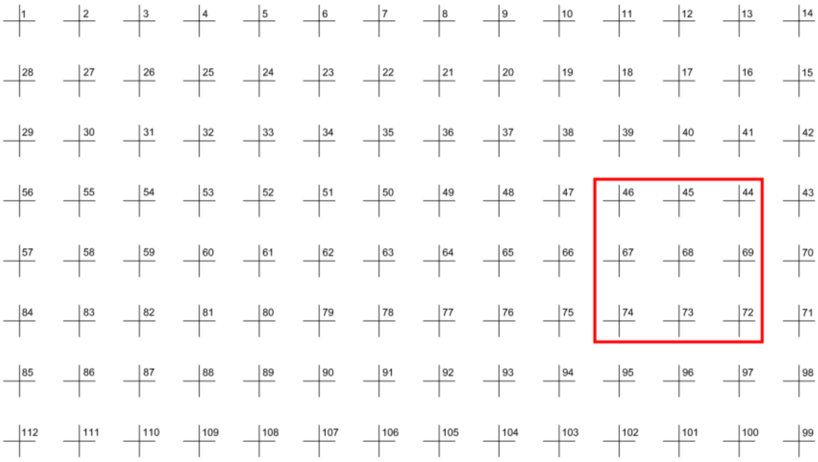

Fig. 2. Grid with marked points on which displacements were simulated (inside a red rectangle).
Source: Own study.

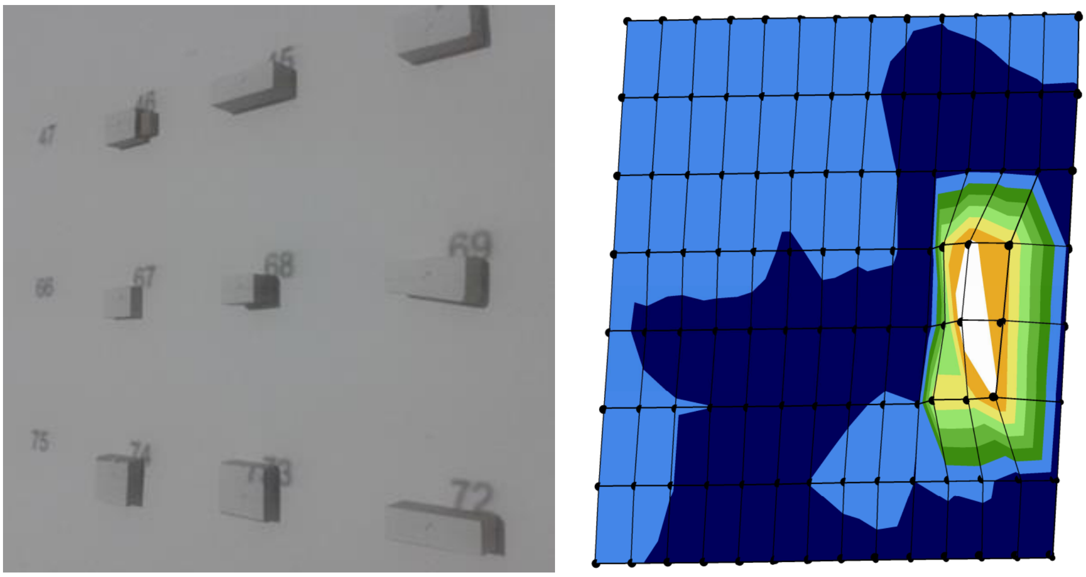

Fig. 3. Wooden blocks simulating displacements (left) and deformation model (right).
Source: Own study.

Table1. Values of simulated displacements.

Point number	Simulated displacement [mm]
44	29
45	33,8
46	22,5
67	17
68	37,5
69	32,3
72	33,5
73	23,2
74	23,4

Source: Own study.

Robust estimation

The initial, i.e. original set of observations and contaminated set was adjusted using classical least squares adjustment and robust estimation methods. Simple method to obtain robust estimates is to use IRLS, i.e. Iteratively Reweighted Least Squares Method (BANAŚ, LIGAS, 2014). The MATLAB software was used to implement IRLS algorithm and individual weighting function of Huber, Hampel, danish and linear method. We can notice that when we apply robust estimation to intentionally contaminated datasets we get more reliable results than from least squares estimation (Tab. 3). The reference to show how M-estimators reduce influence of outliers in contaminated datasets were coordinates obtained from least squares adjustment applied to datasets without contamination. We can notice that classical least squares adjustment is not robust to outliers in dataset.

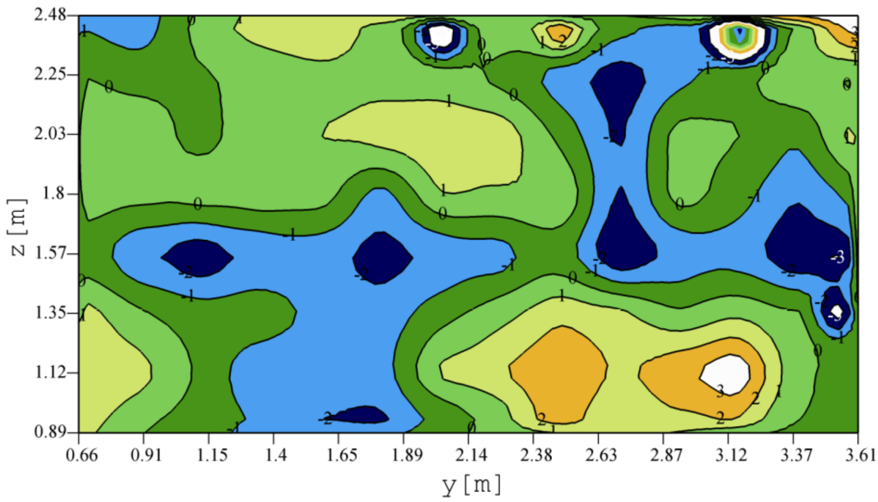

Fig. 4. Contour map of points' deviations from original surface estimated with using least squares method (without simulated outliers).
Source: Own study.

Conclusions

The study presents effect of numerical tests of performance of four robust estimation methods applied to find parameters of flat surface (approximation). Paper presents obtained values of displacements which were intentionally introduced. To estimate them there was used least squares method and 4 robust estimation methods. The paper shows the advantage of robust estimation methods to reduce the influence of outliers on plain surface parameters over the ordinary least squares adjustment. Robust estimation methods allow to estimate parameters of the surface from a set with outliers which are far more close to the parameters estimated with using the original coordinates from measurement of the wall (without contamination). Residuals of the model obtained in points which are free of outliers are smaller than in case of using ordinary least square adjustment for wall with simulated displacements. The model of surface is „closer" to points which are free of blunders than in case of using least square adjustment. Obtained values of simulated displacements as a result of robust estimation are very close to their real values. Each robust method gave very similar results. At the same time, these are results that deviate from those obtained from the least square method (Tab. 3). As the main conclusion, it should be stated that application of robust estimation methods allow to obtain real, local imperfections without spreading big local imperfections to the neighbouring points of the wall.

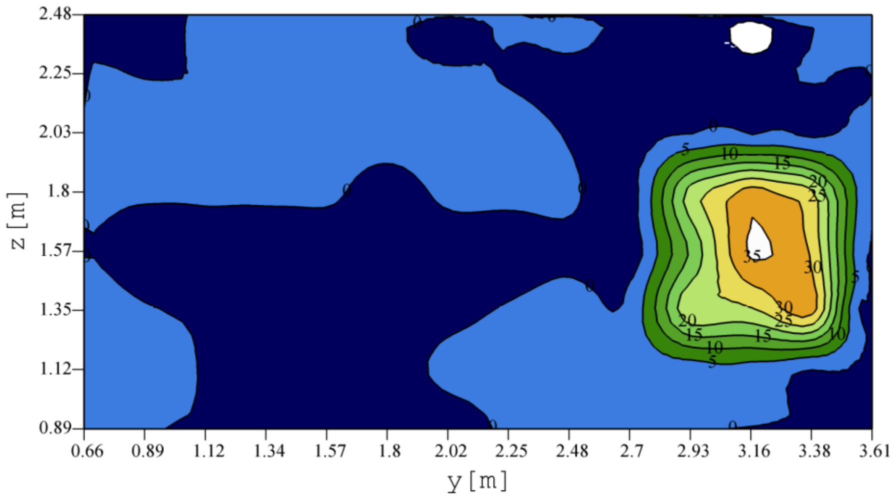

Fig. 5. Contour map of points' deviations (Huber method).
Source: Own study.

Table2. Values of simulated displacements.

Point number	Values of simulated displacements from calliper [mm]	Original wall imperfections [mm]	Values of simulated displacements reduced by original wall imperfections [mm]
44	29,0	-1,9	27,1
45	33,8	-0,7	33,2
46	22,5	0,3	22,8
67	17,0	-1,8	15,2
68	37,5	-1,5	36,0
69	32,3	-2,9	29,4
72	33,5	0,0	33,5
73	23,2	0,7	23,9
74	23,4	0,1	23,5

Source: Own study.

Table3. Values of estimated displacements in simulated points.

Point number	Huber method [mm]	Hampel method [mm]	Danish method [mm]	Linear method [mm]	Least Squares [mm]	Values of simulated displacements reduced by original wall imperfections [mm]
44	27,0	26,9	27,0	26,8	21,4	**27,1**
45	32,9	32,8	32,9	32,7	27,9	**33,2**
46	22,7	22,7	22,8	22,6	18,4	**22,8**
67	15,6	15,6	15,7	15,5	11,1	**15,2**
68	35,9	35,8	35,9	35,7	30,7	**36,0**
69	30,4	30,3	30,4	30,2	24,5	**29,4**
72	33,5	33,5	33,6	33,4	27,6	**33,5**
73	24,1	24,0	24,1	23,9	18,7	**23,9**
74	24,1	24,1	24,1	24,0	19,4	**23,5**

Source: Own study.

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